TABLE I

Substrate Thickness	Open CPW, Static TEM Approximation, Davis [8]		Shielded CPW, Singular Integral Equation, Saha [9]	Odd-mode of Excitation	
	λ'/λ	$Z_0(\Omega)$	λ'/λ	λ'/λ	$Z_0(\Omega)$
Equal to the Slot-width(w)	0.48	57.7	0.443	0.496	55.7
Twice the slot width (2w)	0.45	-	0.439	0.452	51.0
Thrice the slot width (3w)	0.43	52.4	0.436	0.440	49.7

$\epsilon_{r1} = 10$, $\epsilon_{r2} = 1.0$

$s/(2w+s) = 0.5$, $(d_1+d_2+t_2)/d_1 = 9$, $t_1 = 9w$, $b = 9w$

$k_0 = 2\pi f\sqrt{\mu_0 \epsilon_0}$, $k_0 s = 0.02$

and also the characteristic impedance as a function of the normalized substrate thickness with the normalized height of the conducting enclosure as a parameter. It is observed that as the height of the conducting enclosure is gradually reduced, starting from a large value, the even mode slowly cuts off. On the other hand, the odd mode is almost insensitive to the height of the shielding enclosure.

Figs. 2(b) and 3(b) illustrate the computed odd- and even-mode dispersion and also the characteristics impedance, a function of the frequency. The slotlines are separated from each other by an amount $s/(2w+s)$. It is observed that for a fixed frequency and for small values of $s/(2w+s)$ the presence of the conductor between the slots has negligible effect on the even-mode propagation, and the two waves propagate as a single wave on a slotline with slot width $(2w+s)$. Hence the even mode λ'/λ is initially small. When $s/(2w+s)$ is increased, the slot width proportionally increases and the ratio λ'/λ also increases. When $s/(2w+s)$ continues to increase the two waves start to decouple and propagate as two independent waves on two slotlines. These two independent waves finally decouple totally. When each wave propagates on a slotline with slot width w, λ'/λ decreases. Furthermore, the computed even-mode characteristic impedance approaches one-half of the characteristic impedance of a single slotline. The width of the single slotline is twice that of the coupled structure. Nevertheless, the dispersion in both the cases is the same.

If $s/(2w+s)$ is small, and the slots are excited for the odd mode of propagation, then the effective slot width is smaller than w; and hence λ'/λ is the smallest. As the ratio $s/(2w+s)$ increases the effective slot width also increases, consequently λ'/λ increases. Finally, when $s/(2w+s)$ takes a large value the effective slot width moves nearer w; and, the ratio λ'/λ inclines toward the even mode λ'/λ. It may also be noted that the above structures are reduced to a coplanar waveguide (CPW) [8] for the odd mode of excitation. As a numerical check the odd-mode dispersion and characteristic impedance are computed and also compared with the results reported by Davis [8] and Saha [9] in Table I.

IV. Conclusion

Briefly, the paper presents an analysis of shielded coupled slotline a) on a double-layer dielectric substrate, and b) sandwiched between two dielectric substrates. It also illustrates the effect of shielding on the odd- and even-mode dispersion and characteristic impedance. The odd-mode dispersion and characteristic impedance agrees with the results reported by Davis [8] and Saha [9]. These structures should find extensive applications in the design of MIC components, such as, directional couplers, filters, phase shifters, and mixers.

References

[1] S. B. Cohn, "Slot-line on a dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 768–778, Oct. 1969.

[2] B. Schiek, "Hybrid branchline couplers—a useful new class of directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 864–869, Oct. 1974.

[3] M. E. Davis, "Integrated diode phase-shifter elements for an X-band phased-array antenna," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 1080–1084, Dec. 1975.

[4] L. E. Dickens and D. W. Makl, "An integrated-circuit balanced mixer, image and sum enhanced," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 276–281, Mar. 1975.

[5] A. J. Kelly and H. C. Okean, "A low-noise millimeter MIC mixer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 867–869, Nov. 1976.

[6] N. Samardzija and T. Itoh, "Double-layered slot-line for millimeter-wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 827–831, Nov. 1976.

[7] S. B. Cohn, "Sandwich slot-line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 773–774, Sept. 1971.

[8] M. E. Davis, E. W. Williams, and A. C. Celestini, "Finite-boundary corrections to the coplanar waveguide analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 594–596, Sept. 1973.

[9] P. K. Saha, "Dispersion in shielded planar transmission lines on two-layer composite substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 907–911, Nov. 1977.

The Design of Broadside-Coupled Stripline Circuits

I. J. BAHL, MEMBER, IEEE, AND P. BHARTIA, SENIOR MEMBER, IEEE

***Abstract*—Accurate design expressions for broadside-coupled striplines are presented. The expressions have general validity as long as $W/S > 0.35$. Uncertainty analysis is described to calculate the effect of tolerances in parameters on coupling coefficient and input VSWR. The effect of tolerances in parameters on these characteristics increases as the coupling becomes tighter and tighter.**

I. Introduction

Broadside-coupled striplines have been used extensively in the design of many passive and active components, such as directional couplers, filters, baluns, and digital phase shifter networks. This configuration has been used widely for realizing tight couplings (e.g., 3-dB hybrids) because for greater than −8-dB coupling, the spacing between the strips in the case of parallel coupled transmission lines (i.e., striplines, microstrip lines, etc.) becomes prohibitively small.

Broadside-coupled striplines consist of two parallel strip conductors embedded in a dielectric between two ground planes as

Manuscript received August 13, 1979; revised September 8, 1980. This work was supported by the National Research Council of Canada under Grant A-0001.

I. J. Bahl is with the Department of Electrical Engineering, University of Ottawa, Ottawa, Canada K1N 6N5.

P. Bhartia is with the Defence Electronics Division, Defence Research Establishment, Ottawa, Canada, K1A 0Z4.

0018-9480/81/0200-0165$00.75

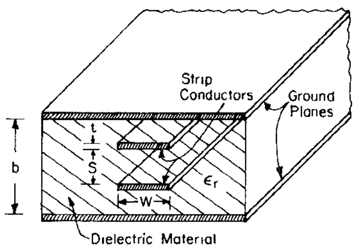


Fig. 1. Broadside-coupled stripline geometry.

shown in Fig. 1. Approximate expressions for the characteristic impedances have been derived by Cohn [1] and Shelton [2], using the conformal mapping technique. In this paper, accurate design expressions for a broadside-coupled line coupler are presented.

II. Design Expressions for a Coupler

Broadside-coupled striplines support two orthogonal TEM modes called even and odd modes. Expressions for the even and odd mode characteristic impedances when the strip thicknesses are negligible are given as follows [1]:

$$Z_{0e}=\frac{60\pi}{\sqrt{\epsilon_r}}\frac{K'(k)}{K(k)} \tag{1}$$

$$Z_{0o}=\frac{296.1}{\sqrt{\epsilon_r}\,\frac{b}{s}\tanh^{-1}(k)} \tag{2}$$

where K is a complete elliptic function of the first kind, and K' is its complementary function given by

$$K'(k)=K(k')=K\left(\sqrt{1-k^2}\right). \tag{3}$$

k is a parameter related to the dimensions of the structure as follows:

$$W/b=\frac{1}{\pi}\left[\ln\left(\frac{1+R}{1-R}\right)-\frac{S}{b}\ln\left(\frac{1+R/k}{1-R/k}\right)\right] \tag{4}$$

where

$$R=\left[\left(k\frac{b}{S}-1\right)\Big/\left(\frac{1}{k}\frac{b}{S}-1\right)\right]^{1/2}$$

and b, W, S are defined in Fig. 1. The results given by (1)–(4) are virtually exact for $W/S \geqslant 0.35$.

If C is the voltage coupling coefficient and Z_0 is the input and output impedance, then Z_{0e} and Z_{0o} can be expressed as [3]

$$Z_{0e}=Z_0\left(\frac{1+C}{1-C}\right)^{1/2} \tag{5}$$

$$Z_{0o}=Z_0\left(\frac{1-C}{1+C}\right)^{1/2} \tag{6}$$

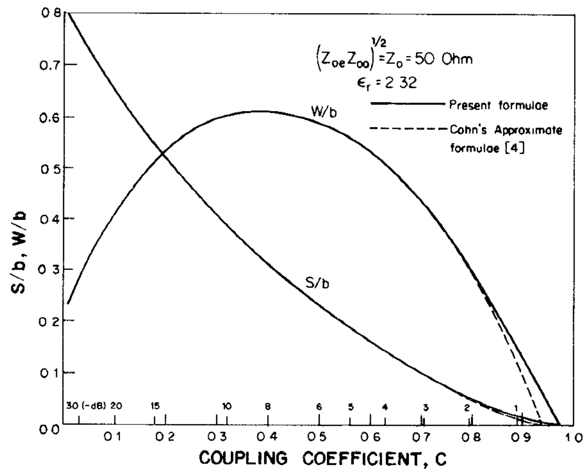
where

$$Z_0=(Z_{0e}Z_{0o})^{1/2}. \tag{7}$$

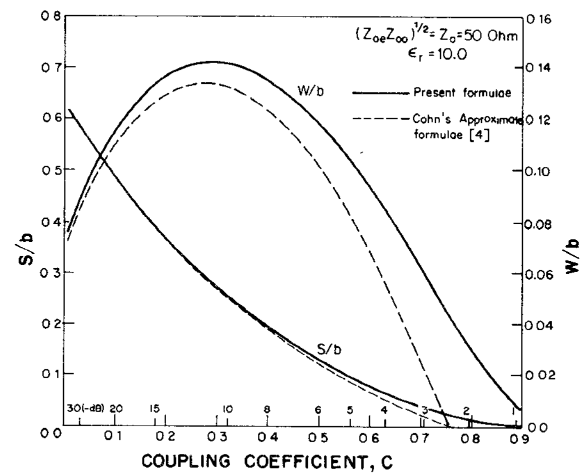
From (1) and (5)

$$\frac{K}{K'}=\frac{60\pi}{Z_0\sqrt{\epsilon_r}}\left(\frac{1-C}{1+C}\right)^{1/2}. \tag{8}$$

The value of the k parameter is required for calculating S/b and W/b from (2) and (4) for the coupled line. Hilberg [4] formulas for K/K' (accurate to 3 ppm) can be used to evaluate k accurately for the given coupling and substrate. These expressions are given as follows:



(a)



(b)

Fig. 2. Variations of S/b and W/b with coupling coefficient. (a) $\epsilon_r=2.32$. (b) $\epsilon_r=10.0$

$$\frac{K}{K'}=\frac{1}{\pi}\ln\left(2\frac{1+\sqrt{k}}{1-\sqrt{k}}\right),\qquad 0.707\leqslant k\leqslant 1 \tag{9a}$$

$$\frac{K}{K'}=\frac{\pi}{\ln\left(2\frac{1+\sqrt{k'}}{1-\sqrt{k'}}\right)},\qquad 0.0\leqslant k\leqslant 0.707. \tag{9b}$$

From (9), for $K/K'\geqslant 1$ or $C\leqslant(1.0-P\epsilon_r)/(1.0+P\epsilon_r)$

$$k=\left(\frac{0.5\exp(\pi K/K')-1}{0.5\exp(\pi K/K')+1}\right)^2. \tag{10a}$$

For $K/K'\leqslant 1$ or $C\geqslant(1.0-P\epsilon_r)/(1.0+P\epsilon_r)$

$$k=\left[1-\left(\frac{0.5\exp(\pi K'/K)-1}{0.5\exp(\pi K'/K)+1}\right)^4\right]^{1/2} \tag{10b}$$

where

$$P=(Z_0/60\pi)^2.$$

From (2) and (6)

$$\frac{S}{b}=0.0017Z_0\sqrt{\epsilon_r}\left(\frac{1-C}{1+C}\right)^{1/2}\ln\left(\frac{1+k}{1-k}\right). \tag{11}$$

For given C, ϵ_r, and Z_0, W/b and S/b required for the design of coupled lines are calculated from (5) through (11). The variations of W/b and S/b as functions of the coupling coefficient for $\epsilon_r=2.32$ and 10.0 are shown in Figs. 2(a) and 2(b), respectively. Approximate results were calculated using expressions given in reference [5] which are obtained by simple manipulation of Cohn's relations [1, eqs. (4)–(7)] or Shelton's expressions [2, with $W_0=0$] and shown in Fig. 2 by the broken lines. It may be observed that for $\epsilon_r=2.32$, the agreement between the two types of results is excellent for $C \leqslant 0.75$. In Fig. 2(b) the S/b curve from approximate results agrees well with the present results for $C \leqslant 0.4$. It should be noted that approximate expressions are valid over the range $(W/b)/(1-S/b) \geqslant 0.35$, and $W/S \geqslant 0.35$, whereas those presented here have general validity as long as $W/S \geqslant 0.35$. Hence, when broadside-coupled stripline couplers are fabricated using high dielectric constant materials, the approximate formulas [1], [2] fail to give correct S/b and W/b values for given coupling coefficient.

III. Uncertainty Analysis

The sensitivity analysis of transmission lines used for MIC's has been treated extensively in the literature [6]–[7]. A more precise method of estimating uncertainty in the characteristics of a circuit due to fabrication tolerances or measurement errors is with the use of uncertainty analysis. In this method, the effect of small error sources becomes negligible in comparison to the larger error sources, since each error term is squared and added to obtain a measure of the total error, whereas in sensitivity analysis all errors are added linearly. If Z is a function of the independent variables $X_1, X_2, \cdots, X_n$ and $\Delta X_1, \Delta X_2, \cdots, \Delta X_n$ are the uncertainties in the independent variables, then the uncertainty in the result, ΔZ, is given by [10]

$$\Delta Z = \pm \left[\sum_{p=1}^{n} \left(\frac{\partial Z}{\partial X_p} \Delta X_p \right)^2 \right]^{1/2}. \tag{12}$$

For broadside-coupled striplines, the expression becomes

$$\Delta Z_{0i} = \pm \left[\left(\frac{\partial Z_{0i}}{\partial W} \Delta W \right)^2 + \left(\frac{\partial Z_{0i}}{\partial S} \Delta S \right)^2 + \left(\frac{\partial Z_{0i}}{\partial b} \Delta b \right)^2 + \left(\frac{\partial Z_{0i}}{\partial t} \Delta t \right)^2 + \left(\frac{\partial Z_{0i}}{\partial \epsilon_r} \Delta \epsilon_r \right)^2 \right]^{1/2} \tag{13}$$

where ΔW, ΔS, Δb, Δt, and $\Delta\epsilon_r$ are the uncertainties in the fabrication of the circuit. The subscript i has two values, e and o, corresponding to even and odd modes, respectively. From (1) and (2)

$$\frac{\partial Z_{0i}}{\partial \epsilon_r} = -\frac{1}{2} \frac{Z_{0i}}{\epsilon_r}. \tag{14}$$

A. Uncertainty in Coupling Coefficient

The maximum coupling coefficient for a coupled line ($l=\lambda/4$) is

$$C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}. \tag{15}$$

Using the uncertainty approach

$$\Delta C = \pm \left[\sum_{p=1}^{5} \left(\frac{\partial C}{\partial X_p} \Delta X_p \right)^2 \right]^{1/2} \tag{16}$$

where the X_p's represent W, S, b, t, and ϵ_r. From (15)

$$\frac{\partial C}{\partial X_p} = 2 \left(Z_{0o} \frac{\partial Z_{0e}}{\partial X_p} - Z_{0e} \frac{\partial Z_{0o}}{\partial X_p} \right) \Big/ (Z_{0e} + Z_{0o})^2. \tag{17}$$

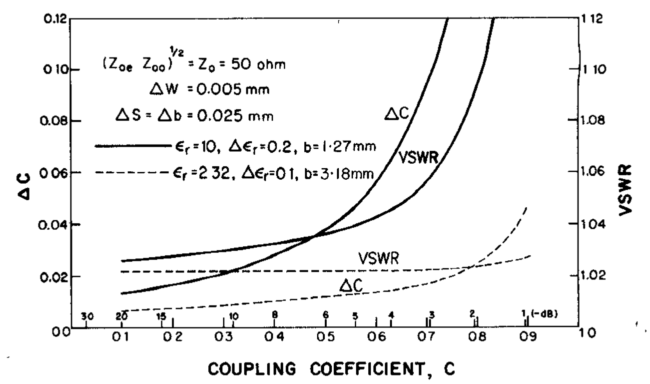


Fig. 3. Fractional change in coupling and VSWR of broadside-coupled stripline couplers caused by manufacturing tolerances in physical dimensions.

Equations (16) and (17) provide the fractional change in C due to fabrication or manufacturing variations. The uncertainty in the coupling coefficient due to manufacturing tolerances as a function of coupling is shown in Fig. 3 for a specified set of parameters. It may be observed from this figure that the uncertainty in coupling coefficient due to a set of tolerances increases with the increase in the value of C. Similar behavior has been noticed for coupled microstrip lines [6], [8].

B. Uncertainty in Input VSWR

The value of VSWR, when measured along an ideal line of characteristic impedance Z_0 connected at the input is given by [9]

$$\mathrm{VSWR} = \left[1 - \frac{|\Delta Z_0|}{Z_0} \right]^{-1} \tag{18}$$

where ΔZ_0 is the uncertainty in Z_0 (which is related to Z_{0e} and Z_{0o} by (7)) due to tolerances in the various parameters and is given by

$$\Delta Z_0 = \pm \left[\sum_{p=1}^{5} \left(\frac{\partial Z_0}{\partial X_p} \Delta X_p \right)^2 \right]^{1/2}. \tag{19}$$

From (7) and (19)

$$\frac{\Delta Z_0}{Z_0} = \pm \frac{1}{2} \left[\sum_{p=1}^{5} \left(\frac{\partial Z_{0e}}{\partial X_p} \frac{\Delta X_p}{Z_{0e}} + \frac{\partial Z_{0o}}{\partial X_p} \frac{\Delta X_p}{Z_{0o}} \right)^2 \right]^{1/2}. \tag{20}$$

Variation of VSWR as a function of coupling coefficient is shown in Fig. 3. Values of VSWR are almost constant with coupling for $\epsilon_r=2.32$ and the VSWR increases with C for $\epsilon_r=10$.

IV. Conclusions

Accurate design expressions for broadside-coupled stripline couplers are presented in this paper. Formulas have general validity as long as $W/S \geqslant 0.35$. Uncertainty analysis has been carried out to calculate the effect of tolerances in parameters on coupling coefficient and input VSWR. It has been found that for a directional coupler with 50-Ω input and output impedance, the change in coupling coefficient, due to tolerances, increases with the increase in the value of C.

References

[1] S. B. Cohn, "Characteristic impedances of broadside-coupled strip transmission lines," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 633–637, Nov. 1960.

[2] J. P. Shelton, Jr., "Impedances of Offset parallel-coupled strip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, pp.

7–15, Jan. 1966.
[3] K. C. Gupta and A. Singh (eds.), *Microwave Integrated Circuits*. New Delhi: Wiley Eastern, 1974, pp. 73–85.
[4] W. Hilberg, "From approximation to exact relations for characteristic impedances," *IEEE Trans. Microwave Theory Tech.*, vol MTT-17, pp. 259–265, 1969.
[5] H. Howe, Jr., *Stripline Circuit Design*. Dedham, MA: Artech House, 1974, p. 125.
[6] R. Garg and I. J. Bahl, "Characteristics of coupled microstrip lines," *IEEE Trans., Microwave Theory Tech.*, vol. MTT-27, pp. 700–705, July 1979.
[7] R. Garg, "The effect of tolerances on microstripline and slotline performance," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 16–19, Jan. 1978.
[8] S. D. Shamasundara and K. C. Gupta, "Sensitivity analysis of coupled microstrip directional couplers," *IEEE Trans. Microwave Theory Tech.*, pp. 788–794, Oct. 1978.
[9] K. C. Gupta, R. Garg and I. J. Bahl, *Microstrip Lines and Slotlines*. Dedham, MA: Artech House, 1979.
[10] J. P. Holman and W. J. Gajda, Jr., *Experimental Methods for Engineers*. New York: McGraw-Hill, 1978 p. 45..

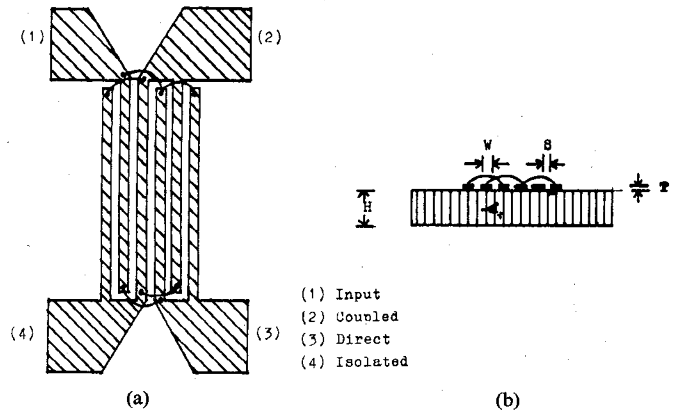


Fig. 1. (a) Conductor pattern and (b) cross section of a six-strip Lange coupler.

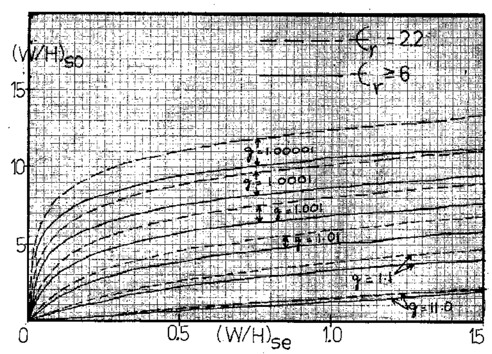


Fig. 2. g as a function of $(W/H)_{so}$ and $(W/H)_{se}$ (4).

Synthesis of Lange Couplers

R. M. OSMANI

***Abstract*—This paper shows that it is possible to synthesize Lange couplers directly and thereby save considerable computing time. The procedure outlined is essentially based on two available techniques: Ou's analysis of the Lange coupler and Akhtarzad's design method for a pair of coupled microstrip lines. Including a correction in the latter for single strip shape ratios less than unity, is significant. The described approach compares favorably with existing iterative methods and was used to obtain reasonably good performances on a low dielectric constant laminate.**

I. Introduction

Tight coupling in microstrip became feasible with the introduction of the Lange coupler [1] in 1969. Recent papers on Lange couplers [2]–[7] are based on an analysis of the coupler and therefore a number of iterations are needed to establish a new design. Analysis is more suitable for confirming or optimizing an initial design [8]; synthesis is preferable for determining the dimensions of a coupler for any given requirement of coupling value, dielectric constant, terminating impedance, and number of strips.

In this paper, a direct synthesis procedure for Lange couplers is outlined. Ou's method is first used to determine odd and even-mode impedances of any adjacent pair of lines in the array [9]. Final dimensions are obtained by applying the synthesis technique of Akhtarzad *et al.* [10]. All equations given here are simple enough to be solved without a computer. Good agreement with more involved analytical methods is shown by both theoretical and experimental results. An empirical correction for finite conductor thickness given by Presser [5] did not explain the extent of overcoupling observed for couplers fabricated on a low dielectric constant substrate.

II. Synthesis

The conductor pattern and cross section of a six-strip Lange coupler is shown in Fig. 1 to illustrate the relevant dimensions. All strips are assumed to be equispaced, to be of equal width, and also to be quarter-wavelength long at the center frequency.

For any adjacent pair of lines in the array, the odd- and even-mode impedances Z_{0o} and Z_{0e}, respectively, are obtained by solving the analysis equations of Ou [9]. Although solutions are available [5], [11] it is shown in Appendix that simpler expressions for Z_{0o} and Z_{0e} are possible:

$$Z_{0o}=Z_0\left(\frac{1-c}{1+c}\right)^{1/2}\cdot\frac{(k-1)\cdot(1+q)}{(c+q)+(k-1)\cdot(1-c)} \tag{1}$$

$$Z_{0e}=Z_{0o}\frac{(c+q)}{(k-1)\cdot(1-c)} \tag{2}$$

where

$$q=\left[c^2+(1-c^2)\cdot(k-1)^2\right]^{1/2}$$

k is the even number of strips, Z_0 is the terminating impedance and c is the voltage coupling coefficient.

The synthesis technique of Akhtarzad *et al.* [10] for a pair of coupled microstriplines can now be applied. In order to relate Z_{0o} and Z_{0e} to the physical dimensions of the coupler, single strip shape ratios $(W/H)_{so}$ and $(W/H)_{se}$ corresponding to the impedances $Z_{0o}/2$ and $Z_{0e}/2$, respectively, are first calculated. In the original method this was done by using Wheeler's equation for wide strips [10]. For shape ratios less than unity, this causes an appreciable error [12]. The procedure adopted here avoids this error by using Wheeler's equation valid for both wide and narrow strips [13]. The required shape ratios $(W/H)_{so}$ and $(W/H)_{se}$ are determined by substituting the known values of

Manuscript received June 20, 1980; revised September 15, 1980.
The author is with Defense Electronics Research Laboratory, Hyderbad-500 005, India.

0018-9480/81/0200-0168$00.75 ©1981 IEEE